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Common fixed points of almost generalized (ψ, ϕ) -contractive mappings in ordered metric spaces

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Abstract

In this article, we introduce the notion of almost generalized (ψ, ϕ) -contractive mappings in ordered metric spaces and we establish some fixed and common fixed point results in ordered complete metric spaces. Our results generalize several well-known comparable results in the literature. Finally, an example and an application are given in order to support the useability of our results.

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1 Introduction

A fundamental principle in computer science is iteration. Iterative techniques are used to find roots of equations, solutions of linear and nonlinear systems of equations, and solution of differential equations. So the attraction of the fixed point iteration is understandable to a large number of mathematicians.

The Banach contraction principle (see [1]) is a very popular tool for solving problems in nonlinear analysis. Some authors generalized this interesting theorem in different ways (see for example [2-20]).

Berinde [21-24] initiated the concept of almost contractions and studied many interesting fixed point theorems for a Ćirić strong almost contraction. So, let us recall the following definition.

Definition 1.1. [21] A single valued mapping $f: X \times X$ is called a Ćirić strong almost contraction if there exist a constant $\alpha \in [0, 1)$ and some $L \geq 0$ such that

$$d(fx, fy) \leq \alpha M(x, y) + Ld(y, fx)$$

for all $x, y \in X$, where

$$M(x, y) = \max \left\{ d(x, y), d(x, fx), d(y, fy), \frac{d(x, fy) + d(y, fx)}{2} \right\}.$$

Babu [25] introduced the class of mappings which satisfy “condition (B)”.

Definition 1.2. [25] Let (X, d) be a metric space. A mapping $f: X \rightarrow X$ is said to satisfy “condition (B)” if there exist a constant $\delta \in (0, 1)$ and some $L \geq 0$ such that

$$d(fx, fy) \leq \delta d(x, y) + L \min \{d(x, fx), d(x, fy), d(y, fx)\},$$

for all $x, y \in X$.

Moreover, Babu proved in [25] the existence of fixed point theorem for such mappings on complete metric spaces.

Ćirić et al. [26] introduced the concept of almost generalized contractive condition and they proved some existing results.

Definition 1.3. [26] Let (X, \preceq) be a partially ordered set. Two mappings $f, g : X \rightarrow X$ are said to be strictly weakly increasing if $fx \prec gfx$ and $gx \prec fgx$, for all $x \in X$.

Definition 1.4. [26] Let f and g be two self mappings on a metric space (X, d) . Then they are said to satisfy almost generalized contractive condition if there exist a constant $\delta \in (0, 1)$ and some $L \geq 0$ such that

$$d(fx, fy) \leq \delta \max \left\{ d(x, y), d(x, fx), d(y, gy), \frac{d(x, gy) + d(y, fx)}{2} \right\} + L \min \{d(x, fx), d(x, gy), d(y, fx)\}, \quad (1)$$

for all $x, y \in X$.

Then Ćirić et al. [26] proved the following theorems.

Theorem 1.1. Let (X, \preceq) be a partially ordered set and suppose that there exists a metric d on X such that the metric space (X, d) is complete. Let $f : X \rightarrow X$ be a strictly increasing continuous mapping with respect to \preceq . Suppose that there exist a constant $\delta \in [0, 1)$ and some $L \geq 0$ such that

$$d(fx, fy) \leq \delta M(x, y) + L \min \{d(x, fx), d(x, fy), d(y, fx)\},$$

for all comparable $x, y \in X$, where

$$M(x, y) = \max \left\{ d(x, y), d(x, fx), d(y, fy), \frac{d(x, fy) + d(y, fx)}{2} \right\}.$$

If there exists $x_0 \in X$ such that $x_0 \preceq fx_0$, then f has a fixed point in X .

Theorem 1.2. Let (X, \preceq) be a partially ordered set and suppose that there exists a metric d on X such that the metric space (X, d) is complete. Let $f, g : X \rightarrow X$ be two strictly weakly increasing mappings which satisfy (1) with respect to \preceq , for all comparable elements $x, y \in X$. If either f or g is continuous, then f and g have a common fixed point in X .

Khan et al. [27] introduced the concept of altering distance function as follows.

Definition 1.5. [27] The function $\phi : [0, +\infty) \rightarrow [0, +\infty)$ is called an altering distance function, if the following properties are satisfied:

- (1) ϕ is continuous and non-decreasing.
- (2) $\phi(t) = 0$ if and only if $t = 0$.

Many authors studied fixed point theorems which are based on altering distance functions, see for example [27-36].

In this article, we introduce the notion of almost generalized (ψ, ϕ) -contractive mapping and we establish some results in complete ordered metric spaces, where ψ and ϕ are altering distance functions. Our results generalize Theorems 1.1 and 1.2.

2 Main results

In this section, we define the notion of almost generalized (ψ, ϕ) -contractive mapping, then we present and prove our new results. In particular, we generalize Theorems 2.1, 2.2 and 2.3 of Ćirić et al. [26].

Let (X, \preceq, d) be an ordered metric space and let $f: X \rightarrow X$ be a mapping. Set

$$M(x, y) = \max \left\{ d(x, y), d(x, fx), d(y, fy), \frac{d(x, fy) + d(y, fx)}{2} \right\}$$

and

$$N(x, y) = \min\{d(x, fx), d(y, fy)\}.$$

Now, we introduce the following definition.

Definition 2.1. Let (X, d) be a metric space and let ψ and ϕ be altering distance functions. We say that a mapping $f: X \rightarrow X$ is an almost generalized (ψ, ϕ) -contractive mapping if there exists $L \geq 0$ such that

$$\psi(d(fx, fy)) \leq \psi(M(x, y)) - \phi(M(x, y)) + L\psi(N(x, y)) \quad (2)$$

for all comparable $x, y \in X$.

Throughout this article, the mappings ψ and ϕ denote altering distance functions. Now, let us prove our first result.

Theorem 2.1. Let (X, \preceq) be a partially ordered set and suppose that there exists a metric d on X such that the metric (X, d) is complete. Let $f: X \rightarrow X$ be a non-decreasing continuous mapping with respect to \preceq . Suppose that f is an almost generalized (ψ, ϕ) -contractive mapping. If there exists $x_0 \in X$ such that $x_0 \preceq fx_0$, then f has a fixed point.

Proof. Let $x_0 \in X$. Then, we define a sequences (x_n) in X such that $x_{n+1} = fx_n$. Since $x_0 \preceq fx_0 = x_1$ and f is non-decreasing, we have $x_1 = fx_0 \preceq x_2 = fx_1$. Again, as $x_1 \preceq x_2$ and f is non-decreasing, we have $x_2 = fx_1 \preceq x_3 = fx_2$. By induction, we show that

$$x_0 \preceq x_1 \preceq \dots \preceq x_n \preceq x_{n+1} \preceq \dots$$

If $x_n = x_{n+1}$ for some $n \in \mathbb{N}$, then $x_n = fx_n$ and hence x_n is a fixed point of f . So, we may assume that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$. By (2), we have

$$\begin{aligned} \psi(d(x_n, x_{n+1})) &= \psi(d(fx_{n-1}, fx_n)) \\ &\leq \psi(M(x_{n-1}, x_n)) - \phi(M(x_{n-1}, x_n)) + L\psi(N(x_{n-1}, x_n)), \end{aligned} \quad (3)$$

where

$$\begin{aligned} M(x_{n-1}, x_n) &= \max \left\{ d(x_{n-1}, x_n), d(x_{n-1}, fx_{n-1}), d(x_n, fx_n), \frac{d(x_{n-1}, fx_n) + d(x_n, fx_{n-1})}{2} \right\} \\ &= \max \left\{ d(x_{n-1}, x_n), d(x_n, x_{n+1}), \frac{d(x_{n-1}, x_{n+1})}{2} \right\} \\ &\leq \max \left\{ d(x_{n-1}, x_n), d(x_n, x_{n+1}), \frac{d(x_{n-1}, x_n) + d(x_n, x_{n+1})}{2} \right\} \\ &= \max \{d(x_{n-1}, x_n), d(x_n, x_{n+1})\} \end{aligned} \quad (4)$$

and

$$\begin{aligned} N(x_{n-1}, x_n) &= \min\{d(x_{n-1}, fx_{n-1}), d(x_n, fx_n)\} \\ &= \min\{d(x_{n-1}, x_n), 0\} \\ &= 0. \end{aligned} \quad (5)$$

From (3)-(5) and the properties of ψ and ϕ , we get

$$\begin{aligned} \psi(d(x_n, x_{n+1})) &\leq \psi(\max\{d(x_{n-1}, x_n), d(x_n, x_{n+1})\}) - \phi(\max\{d(x_{n-1}, x_n), d(x_n, x_{n+1})\}) \\ &\leq \psi(\max\{d(x_{n-1}, x_n), d(x_n, x_{n+1})\}) \end{aligned} \quad (6)$$

If

$$\max\{d(x_{n-1}, x_n), d(x_n, x_{n+1})\} = d(x_n, x_{n+1}),$$

then by (6) we have

$$\begin{aligned}\psi(d(x_n, x_{n+1})) &\leq \psi(d(x_n, x_{n+1})) - \phi(d(x_n, x_{n+1})) \\ &< \psi(d(x_n, x_{n+1})),\end{aligned}$$

which gives a contradiction. Thus,

$$\max\{d(x_{n-1}, x_n), d(x_n, x_{n+1})\} = d(x_{n-1}, x_n).$$

Therefore (6) becomes

$$\psi(d(x_n, x_{n+1})) \leq \psi(d(x_n, x_{n-1})) - \phi(d(x_{n-1}, x_n)) < \psi(d(x_n, x_{n-1})). \quad (7)$$

Since ψ is a non-decreasing mapping, we have $\{d(x_n, x_{n+1}): n \in \mathbb{N} \cup \{0\}\}$ is a non-increasing sequence of positive numbers. So, there exists $r \geq 0$ such that

$$\lim_{n \rightarrow +\infty} d(x_n, x_{n+1}) = r.$$

Letting $n \rightarrow +\infty$ in (7), we get

$$\psi(r) \leq \psi(r) - \phi(r) \leq \psi(r).$$

Therefore $\phi(r) = 0$, and hence $r = 0$. Thus, we have

$$\lim_{n \rightarrow +\infty} d(x_n, x_{n+1}) = 0. \quad (8)$$

Next, we show that (x_n) is a Cauchy sequence in X . Suppose to the contrary; that is, (x_n) is not a Cauchy sequence. Then there exists $\varepsilon > 0$ for which we can find two subsequences $(x_{m(i)})$ and $(x_{n(i)})$ of (x_n) such that $n(i)$ is the smallest index for which

$$n(i) > m(i) > i, \quad d(x_{m(i)}, x_{n(i)}) \geq \varepsilon. \quad (9)$$

This means that

$$d(x_{m(i)}, x_{n(i)-1}) < \varepsilon. \quad (10)$$

From (9), (10) and the triangular inequality, we get

$$\begin{aligned}\varepsilon &\leq d(x_{m(i)}, x_{n(i)}) \\ &\leq d(x_{m(i)}, x_{m(i)-1}) + d(x_{m(i)-1}, x_{n(i)}) \\ &\leq d(x_{m(i)}, x_{m(i)-1}) + d(x_{m(i)-1}, x_{n(i)-1}) + d(x_{n(i)-1}, x_{n(i)}) \\ &\leq 2d(x_{m(i)}, x_{m(i)-1}) + d(x_{m(i)}, x_{n(i)-1}) + d(x_{n(i)-1}, x_{n(i)}) \\ &< 2d(x_{m(i)}, x_{m(i)-1}) + \varepsilon + d(x_{n(i)-1}, x_{n(i)}).\end{aligned}$$

Using (8) and letting $i \rightarrow +\infty$, we get

$$\begin{aligned}\lim_{i \rightarrow +\infty} d(x_{m(i)}, x_{n(i)}) &= \lim_{i \rightarrow +\infty} d(x_{m(i)-1}, x_{n(i)}) \\ &= d(x_{m(i)}, x_{n(i)-1}) \\ &= d(x_{m(i)-1}, x_{n(i)-1}) \\ &= \varepsilon.\end{aligned} \quad (11)$$

From (2), we have

$$\begin{aligned} \psi(d(x_{m(i)}, x_{n(i)})) &= \psi(d(fx_{m(i)-1}, fx_{n(i)-1})) \\ &\leq \psi(M(x_{m(i)-1}, x_{n(i)-1})) - \phi(M(x_{m(i)-1}, x_{n(i)-1})) + L\psi(N(x_{m(i)-1}, x_{n(i)-1})), \end{aligned} \quad (12)$$

where

$$\begin{aligned} M(x_{m(i)-1}, x_{n(i)-1}) &= \max \left\{ d(x_{m(i)-1}, x_{n(i)-1}), d(x_{m(i)-1}, fx_{m(i)-1}), d(x_{n(i)-1}, fx_{n(i)-1}), \right. \\ &\quad \left. \frac{d(x_{m(i)-1}, fx_{n(i)-1}) + d(fx_{m(i)-1}, x_{n(i)-1})}{2} \right\} \\ &= \max \left\{ d(x_{m(i)-1}, x_{n(i)-1}), d(x_{m(i)-1}, x_{m(i)}), d(x_{n(i)-1}, x_{n(i)}), \right. \\ &\quad \left. \frac{d(x_{m(i)-1}, x_{n(i)}) + M(x_{m(i)}, x_{n(i)-1})}{2} \right\} \end{aligned} \quad (13)$$

and

$$\begin{aligned} N(x_{m(i)-1}, x_{n(i)-1}) &= \min \{ d(x_{m(i)-1}, fx_{m(i)-1}), d(fx_{m(i)-1}, x_{n(i)-1}) \} \\ &= \min \{ d(x_{m(i)-1}, x_{m(i)}), d(x_{m(i)}, x_{n(i)-1}) \}. \end{aligned} \quad (14)$$

Letting $i \rightarrow +\infty$ in (13) and (14) then using (8) and (11), we get

$$\lim_{i \rightarrow +\infty} M(x_{m(i)-1}, x_{n(i)-1}) = \varepsilon \quad (15)$$

and

$$\lim_{i \rightarrow +\infty} N(x_{m(i)-1}, x_{n(i)-1}) = 0. \quad (16)$$

Letting $i \rightarrow +\infty$ in (12) then using (11), (15) and (16) we have

$$\psi(\varepsilon) \leq \psi(\varepsilon) - \phi(\varepsilon) < \psi(\varepsilon),$$

which gives a contradiction. Thus $(x_{n+1} = fx_n)$ is a Cauchy sequence in X . As X is a complete space, there exists $u \in X$ such that

$$\lim_{n \rightarrow +\infty} x_{n+1} = \lim_{n \rightarrow +\infty} fx_n = u.$$

Now, suppose that f is continuous, then $fx_n \rightarrow fu$. By the uniqueness of limit, we have $fu = u$. Thus u is a fixed point of f . \square

Notice that the continuity of f in Theorem 2.1 is not necessary and can be dropped.

Theorem 2.2. *Under the same hypotheses of Theorem 2.1 and without assuming the continuity of f , assume that whenever (x_n) is a non-decreasing sequence in X such that $x_n \rightarrow x \in X$ implies $x_n \preceq x$, for all $n \in \mathbb{N}$. Then f has a fixed point in X .*

Proof. Following similar arguments to those given in Theorem 2.1, we construct an increasing sequence (x_n) in X such that $x_n \rightarrow u$ for some $u \in X$. Using the assumption of X , we have $x_n \preceq u$ for all $n \in \mathbb{N}$. Now, we show that $fu = u$. By (2), we have

$$\begin{aligned} \psi(d(x_{n+1}, fu)) &= \psi(d(fx_n, fu)) \\ &\leq \psi(M(x_n, u)) - \phi(M(x_n, u)) + L\psi(N(x_n, u)), \end{aligned} \quad (17)$$

where

$$\begin{aligned} M(x_n, u) &= \max \left\{ d(x_n, u), d(x_n, fx_n), d(u, fu), \frac{d(x_n, fu) + d(fx_n, u)}{2} \right\} \\ &= \max \left\{ d(x_n, u), d(x_n, x_{n+1}), d(u, fu), \frac{d(x_n, fu) + d(x_{n+1}, u)}{2} \right\} \end{aligned} \quad (18)$$

and

$$\begin{aligned} N(x_n, u) &= \min\{d(x_n, fx_n), d(u, fx_n)\} \\ &= \min\{d(x_n, x_{n+1}), d(u, x_{n+1})\}. \end{aligned} \quad (19)$$

Letting $n \rightarrow +\infty$ in (18) and (19) then we get $M(x_n, u) \rightarrow d(u, fu)$ and $N(x_n, u) \rightarrow 0$. Again when $n \rightarrow +\infty$ in (17) then we get

$$\psi(d(u, fu)) \leq \psi(d(u, fu)) - \phi(d(u, fu)).$$

Therefore, $d(u, fu) = 0$. Thus $u = fu$ and hence u is a fixed point of f . \square

Corollary 2.1. *Let (X, \preceq) be a partially ordered set and suppose that there exists a metric d on X such that the metric (X, d) is complete. Let $f: X \rightarrow X$ be a non-decreasing continuous mapping with respect to \preceq . Suppose that there exist $k \in [0, 1)$ and $L \geq 0$ such that*

$$d(fx, fy) \leq k \max \left\{ d(x, y), d(x, fx), d(y, fy), \frac{d(x, fy) + d(y, fx)}{2} \right\} + L \min\{d(x, fx), d(y, fy)\}$$

for all comparable $x, y \in X$. If there exists $x_0 \in X$ such that $x_0 \leq fx_0$, then f has a fixed point.

Proof. Follows from Theorem (2.1) by taking $\psi(t) = t$ and $\phi(t) = (1-k)t$ for all $t \in [0, +\infty)$ and noticing that f is an almost generalized (ψ, ϕ) -contractive mapping. \square

The continuity of f in Corollary 2.1 is not necessary and can be dropped.

Corollary 2.2. *Under the hypotheses of Corollary 2.1 and without assuming the continuity of f , assume that whenever (x_n) is a non-decreasing sequence in X such that $x_n \rightarrow x \in X$ implies $x_n \preceq x$ for all $n \in \mathbb{N}$. Then f has a fixed point in X .*

Proof. Follows from Theorem (2.2) by taking $\psi(t) = t$ and $\phi(t) = (1-k)t$ for all $t \in [0, +\infty)$. \square

Now, let (X, \preceq, d) be an ordered metric space and let $f, g: X \rightarrow X$ be two mappings. Set

$$M(x, y) = \max \left\{ d(x, y), d(x, fx), d(y, gy), \frac{d(x, gy) + d(y, fx)}{2} \right\}$$

and

$$N(x, y) = \min\{d(x, fx), d(y, gy), d(x, gy)\}.$$

Then we introduce the following definition.

Definition 2.2. *Let (X, \preceq) be a partially ordered set having a metric d , and let ψ and ϕ be altering distance functions. We say that a mapping $f: X \rightarrow X$ is an almost generalized (ψ, ϕ) -contractive mapping with respect to a mapping $g: X \rightarrow X$ if there exists $L \geq 0$ such that*

$$\psi(d(fx, gy)) \leq \psi(M(x, y)) - \phi(M(x, y)) + L\psi(N(x, y)) \quad (20)$$

for all comparable $x, y \in X$.

Definition 2.3. *Let (X, \preceq) be a partially ordered set. Then two mappings $f, g: X \rightarrow X$ are said to be weakly increasing if $fx \preceq gfx$ and $gx \preceq fgx$, for all $x \in X$.*

Note that every strictly weakly increasing mapping is weakly increasing.

Theorem 2.3. *Let (X, \preceq) be a partially ordered set and suppose that there exists a metric d on X such that the metric (X, d) is complete, and let $f, g: X \rightarrow X$ be two*

weakly increasing mappings with respect to \preceq . Suppose that f is an almost generalized (ψ, ϕ) -contractive mapping with respect to g . If either f or g is continuous, then f and g have a common fixed point.

Proof. Let us divide the proof into two parts:

First part: We prove that u is a fixed point of f if and only if u is a fixed point of g . Now, suppose that u is a fixed point of f , then $fu = u$. As $u \preceq u$, by (20), we have

$$\begin{aligned}\psi(d(u, gu)) &= \psi(d(fu, gu)) \\ &\leq \psi\left(\max\left\{d(u, fu), d(u, gu), \frac{1}{2}(d(u, gu) + d(u, fu))\right\}\right) \\ &\quad - \phi\left(\max\left\{d(u, fu), d(u, gu), \frac{1}{2}(d(u, gu) + d(u, fu))\right\}\right) + L \min\{d(u, fu), d(u, gu)\} \\ &= \psi(d(u, gu)) - \phi(d(u, gu)).\end{aligned}$$

Thus we have $\phi(d(u, gu)) = 0$. Therefore $d(u, gu) = 0$ and hence $gu = u$. Similarly, we show that if u is a fixed point of g , then u is a fixed point of f .

Second part (construction a sequence by iterative technique):

Let $x_0 \in X$. We construct a sequence (x_n) in X such that $x_{2n+1} = fx_{2n}$ and $x_{2n+2} = gx_{2n+1}$, for all non-negative integers, i.e. $n \in \mathbb{N} \cup \{0\}$. As f and g are weakly increasing with respect to \preceq , we obtain the following:

$$x_1 = fx_0 \preceq gfx_0 = x_2 = gx_1 \preceq fgx_1 = x_3 \preceq \dots x_{2n+1} = fx_{2n} \preceq gfx_{2n} = x_{2n+2} \preceq \dots$$

If $x_{2n} = x_{2n+1}$ for some $n \in \mathbb{N}$, then $x_{2n} = fx_{2n}$. Thus x_{2n} is a fixed point of f . By the first part, we conclude that x_{2n} is also a fixed point of g .

If $x_{2n+1} = x_{2n+2}$ for some $n \in \mathbb{N}$, then $x_{2n+1} = gx_{2n+1}$. Thus x_{2n+1} is a fixed point of g . By the first part, we conclude that x_{2n+1} is also a fixed point of f .

Therefore, we may assume that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$. Now we complete the proof in the following steps:

Step 1. We will prove that

$$\lim_{n \rightarrow +\infty} d(x_n, x_{n+1}) = 0.$$

As x_{2n+1} and x_{2n+2} are comparable, by (20), we have

$$\begin{aligned}\psi(d(x_{2n+1}, x_{2n+2})) &= \psi(d(fx_{2n}, gx_{2n+1})) \\ &\leq \psi(M(x_{2n}, x_{2n+1})) - \phi(M(x_{2n}, x_{2n+1})) + L\psi(N(x_{2n}, x_{2n+1})),\end{aligned}$$

where

$$\begin{aligned}M(x_{2n}, x_{2n+1}) &= \max\left\{d(x_{2n}, x_{2n+1}), d(x_{2n}, fx_{2n}), d(x_{2n+1}, gx_{2n+1}), \frac{d(fx_{2n}, x_{2n+1}) + d(x_{2n}, gx_{2n+1})}{2}\right\} \\ &= \max\left\{d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2}), \frac{d(x_{2n}, x_{2n+2})}{2}\right\} \\ &\leq \max\{d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2})\},\end{aligned}$$

and

$$\begin{aligned}N(x_{2n}, x_{2n+1}) &= \min\{d(x_{2n}, fx_{2n}), d(x_{2n+1}, fx_{2n}), d(x_{2n}, gx_{2n+1})\} \\ &= \min\{d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+1}), d(x_{2n}, x_{2n+2})\} \\ &= 0.\end{aligned}$$

So, we have

$$\begin{aligned} \psi(d(x_{2n+1}, x_{2n+2})) &\leq \psi(\max\{d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2})\}) \\ &\quad - \phi(\max\{d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2})\}) \end{aligned} \quad (21)$$

If

$$\max\{d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2})\} = d(x_{2n+1}, x_{2n+2}),$$

then (21) becomes

$$\psi(d(x_{2n+1}, x_{2n+2})) \leq \psi(d(x_{2n+1}, x_{2n+2})) - \phi(d(x_{2n+1}, x_{2n+2})) < \psi(d(x_{2n+1}, x_{2n+2})),$$

which gives a contradiction. So

$$\max\{d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2})\} = d(x_{2n}, x_{2n+1}),$$

and hence (21) becomes

$$\psi(d(x_{2n+1}, x_{2n+2})) \leq \psi(d(x_{2n}, x_{2n+1})) - \phi(d(x_{2n}, x_{2n+1})) \leq \psi(d(x_{2n}, x_{2n+1})). \quad (22)$$

Similarly, we show that

$$\psi(d(x_{2n+1}, x_{2n})) \leq \psi(d(x_{2n-1}, x_{2n})) - \phi(d(x_{2n-1}, x_{2n})) \leq \psi(d(x_{2n-1}, x_{2n})). \quad (23)$$

By (22) and (23), we get that $\{d(x_n, x_{n+1}); n \in \mathbb{N}\}$ is a non-increasing sequence of positive numbers. Hence there is $r \geq 0$ such that

$$\lim_{n \rightarrow +\infty} d(x_n, x_{n+1}) = r.$$

Letting $n \rightarrow +\infty$ in (22), we get

$$\psi(r) \leq \psi(r) - \phi(r) \leq \psi(r),$$

which implies that $\phi(r) = 0$ and hence $r = 0$.

So, we have

$$\lim_{n \rightarrow +\infty} d(x_n, x_{n+1}) = 0. \quad (24)$$

Step 2. We will prove that (x_n) is a Cauchy sequence. It is sufficient to show that (x_{2n}) is a Cauchy sequence. Suppose to the contrary; that is, (x_{2n}) is not a Cauchy sequence. Then there exists $\varepsilon > 0$ for which we can find two subsequences of positive integers $(x_{2m(i)})$ and $(x_{2n(i)})$ such that $n(i)$ is the smallest index for which

$$n(i) > m(i) > i, \quad d(x_{2m(i)}, x_{2n(i)}) \geq \varepsilon. \quad (25)$$

This means that

$$d(x_{2m(i)}, x_{2n(i)-2}) < \varepsilon. \quad (26)$$

From (25), (26) and the triangular inequality, we get

$$\begin{aligned} \varepsilon &\leq d(x_{2m(i)}, x_{2n(i)}) \\ &\leq d(x_{2m(i)}, x_{2n(i)-2}) + d(x_{2n(i)-2}, x_{2n(i)-1}) + d(x_{2n(i)-1}, x_{2n(i)}) \\ &< \varepsilon + d(x_{2n(i)-2}, x_{2n(i)-1}) + d(x_{2n(i)-1}, x_{2n(i)}). \end{aligned}$$

By letting $i \rightarrow +\infty$ in the above inequality and using (24), we have

$$\lim_{i \rightarrow +\infty} d(x_{2m(i)}, x_{2n(i)}) = \varepsilon. \quad (27)$$

Moreover,

$$\begin{aligned} \varepsilon &\leq d(x_{2m(i)}, x_{2n(i)}) \\ &\leq d(x_{2m(i)}, x_{2m(i)+1}) + d(x_{2m(i)+1}, x_{2n(i)+1}) + d(x_{2n(i)+1}, x_{2n(i)}) \\ &\leq d(x_{2m(i)}, x_{2m(i)+1}) + d(x_{2m(i)+1}, x_{2n(i)}) + 2d(x_{2n(i)+1}, x_{2n(i)}) \\ &\leq d(x_{2m(i)}, x_{2m(i)+1}) + d(x_{2m(i)+1}, x_{2m(i)+2}) + d(x_{2m(i)+2}, x_{2n(i)}) + 2d(x_{2n(i)+1}, x_{2n(i)}) \\ &\leq 2d(x_{2m(i)}, x_{2m(i)+1}) + 2d(x_{2m(i)+1}, x_{2m(i)+2}) + d(x_{2m(i)}, x_{2n(i)}) + 2d(x_{2n(i)+1}, x_{2n(i)}). \end{aligned}$$

Using (24), (27) and letting $i \rightarrow +\infty$, we get

$$\begin{aligned} \lim_{i \rightarrow +\infty} d(x_{2m(i)}, x_{2n(i)}) &= \lim_{i \rightarrow +\infty} d(x_{2m(i)+1}, x_{2n(i)+1}) \\ &= \lim_{i \rightarrow +\infty} d(x_{2m(i)+1}, x_{2n(i)}) \\ &= \lim_{i \rightarrow +\infty} d(x_{2m(i)+2}, x_{2n(i)}) = \varepsilon. \end{aligned} \quad (28)$$

Since $x_{2n(i)}$ and $x_{2m(i)+1}$ are comparable, so by (20) we have

$$\begin{aligned} \psi(d(x_{2n(i)+1}, x_{2m(i)+2})) &= \psi(d(fx_{2n(i)}, gx_{2m(i)+1})) \\ &\leq \psi(M(x_{2n(i)}, x_{2m(i)+1})) - \phi(M(x_{2n(i)}, x_{2m(i)+1})) + L\psi(N(x_{2n(i)}, x_{2m(i)+1})), \end{aligned} \quad (29)$$

where

$$\begin{aligned} M(x_{2n(i)}, x_{2m(i)+1}) &= \max \left\{ d(x_{2n(i)}, x_{2m(i)+1}), d(x_{2n(i)}, fx_{2n(i)}), d(x_{2m(i)+1}, gx_{2m(i)+1}), \right. \\ &\quad \left. \frac{d(fx_{2n(i)}, x_{2m(i)+1}) + d(x_{2n(i)}, gx_{2m(i)+1})}{2} \right\} \\ &= \max \left\{ d(x_{2n(i)}, x_{2m(i)+1}), d(x_{2n(i)}, x_{2n(i)+1}), d(x_{2m(i)+1}, x_{2m(i)+2}), \right. \\ &\quad \left. \frac{d(x_{2n(i)+1}, x_{2m(i)+1}) + d(x_{2n(i)}, x_{2m(i)+2})}{2} \right\} \end{aligned} \quad (30)$$

and

$$\begin{aligned} N(x_{2n(i)}, x_{2m(i)+1}) &= \max\{d(x_{2n(i)}, fx_{2n(i)}), d(x_{2m(i)+1}, fx_{2n(i)}), d(x_{2n(i)}, gx_{2m(i)+1})\} \\ &= \max\{d(x_{2n(i)}, x_{2n(i)+1}), d(x_{2m(i)+1}, x_{2n(i)+1}), d(x_{2n(i)}, x_{2m(i)+2})\} \\ &= 0. \end{aligned} \quad (31)$$

By letting $i \rightarrow +\infty$ in (30) and (31), we get

$$\lim_{i \rightarrow +\infty} M(x_{2n(i)}, x_{2m(i)+1}) = \varepsilon.$$

Now, letting $i \rightarrow +\infty$ in (29) we get

$$\psi(\varepsilon) \leq \psi(\varepsilon) - \phi(\varepsilon).$$

So $\phi(\varepsilon) = 0$ and then $\varepsilon = 0$, which is a contradiction. Hence (x_n) is a Cauchy sequence in X .

Step 3. (A common fixed point)

As (x_n) is a Cauchy sequence in X which is a complete space, there exists $u \in X$ such that $x_n \rightarrow u$ as $n \rightarrow +\infty$. Without loss of generality, we may assume that f is continuous. Since $x_{2n} \rightarrow u$ as $n \rightarrow +\infty$, we have $x_{2n+1} = fx_{2n} \rightarrow fu$ as $n \rightarrow +\infty$. By the uniqueness of limit we get $fu = u$. Thus u is a fixed point of f . By the first part, we conclude that u is also a fixed point of g . \square

The continuity of one of the functions f or g in Theorem 2.3 is not necessary and can be dropped.

Theorem 2.4. *Under the hypotheses of Theorem 2.3 and without assuming the continuity of one of the functions f or g , assume that whenever (x_n) is a non-decreasing sequence in X such that $x_n \rightarrow x \in X$ implies $x_n \preceq x$, for all $n \in \mathbb{N}$. Then f and g have a common fixed point in X .*

Proof. Following similar arguments to those given in Theorem 2.3, we construct an increasing sequence (x_n) in X such that $x_n \rightarrow u$ for some $u \in X$. Using the assumption on X , we have $x_n \preceq u$ for all $n \in \mathbb{N}$. Now, we show that $fu = gu = u$. By (2), we have

$$\begin{aligned}\psi(d(x_{2n+1}, gu)) &= \psi(d(fx_{2n}, gu)) \\ &\leq \psi(M(x_{2n}, u)) - \phi(M(x_{2n}, u)) + L\psi(N(x_{2n}, u)),\end{aligned}\quad (32)$$

where

$$\begin{aligned}M(x_{2n}, u) &= \max \left\{ d(x_{2n}, u), d(x_{2n}, fx_{2n}), d(u, gu), \frac{d(x_{2n}, gu) + d(fx_{2n}, u)}{2} \right\} \\ &= \max \left\{ d(x_{2n}, u), d(x_{2n}, x_{2n+1}), d(u, gu), \frac{d(x_{2n}, gu) + d(x_{2n+1}, u)}{2} \right\}\end{aligned}\quad (33)$$

and

$$\begin{aligned}N(x_{2n}, u) &= \min\{d(x_{2n}, fx_{2n}), d(u, fx_{2n}), d(x_{2n}, gu)\} \\ &= \min\{d(x_{2n}, x_{2n+1}), d(u, x_{2n+1}), d(x_{2n}, gu)\}.\end{aligned}\quad (34)$$

Letting $n \rightarrow +\infty$ in (33) and (34) then we get $M(x_{2n}, u) \rightarrow d(u, gu)$ and $N(x_{2n}, u) \rightarrow 0$ as $n \rightarrow +\infty$. Again when $n \rightarrow +\infty$ in (32), we get

$$\psi(d(u, gu)) \leq \psi(d(u, gu)) - \phi(d(u, gu)).$$

Therefore, $d(u, gu) = 0$ thus $u = gu$ and then u is a fixed point of g . Similarly, we may show that $fu = u$. Hence u is a common fixed point of f and g . \square

Then we have the following consequence results.

Corollary 2.3. *Let (X, \preceq) be a partially ordered set and suppose that there exists a metric d on X such that the metric (X, d) is complete. Let $f, g : X \rightarrow X$ be two strictly weakly increasing mappings with respect to \preceq . Suppose that there exist $k \in [0, 1)$ and $L \geq 0$ such that*

$$d(fx, gy) \leq k \max \left\{ d(x, y), d(x, fx), d(y, gy), \frac{d(x, gy) + d(fx, y)}{2} \right\} + L \min\{d(x, fx), d(y, gy), d(x, y)\}$$

for all comparable $x, y \in X$. If either f or g is continuous, then f and g have a common fixed point.

Proof. Define $\psi, \phi : [0, +\infty) \rightarrow [0, +\infty)$ by $\psi(t) = t$ and $\phi(t) = (1-k)t$. Then f is an almost generalized (ψ, ϕ) -contractive mapping with respect to g . The proof follows from Theorem 2.3. \square

The continuity of one of the functions f or g in Corollary 2.3 is not necessary and can be dropped.

Corollary 2.4. *Under the hypotheses of Corollary 2.3 and without assuming the continuity of one of the functions f or g , assume that whenever (x_n) is a non-decreasing sequence in X such that $x_n \rightarrow x \in X$ implies $x_n \preceq x$, for all $n \in \mathbb{N}$. Then f and g have a common fixed point in X .*

Proof. Follows from Theorem 2.4. \square

Now, in order to support the useability of our results, let us introduce the following example.

Example 2.1. Let $\times = \{0, 1, 2, \dots\}$. Define the function $f, g : X \rightarrow X$ by

$$fx = \begin{cases} 0, & x = 0; \\ x - 1, & x \neq 0, \end{cases}$$

and

$$gx = \begin{cases} 0, & x \in \{0, 1\}; \\ x - 2, & x \geq 2. \end{cases}$$

Let $d : X \times X \rightarrow \mathbb{R}^+$ be given by

$$d(x, y) = \begin{cases} 0, & x = y; \\ x + y, & x \neq y. \end{cases}$$

Define $\psi, \phi : [0, +\infty) \rightarrow [0, +\infty)$ by $\psi(t) = t^2$ and $\phi(t) = t$. Define a relation \preccurlyeq on X by $x \preccurlyeq y$ iff $y \leq x$. Then we have the following:

(1) (X, \preccurlyeq) is a partially ordered set having the metric d , where the metric space (X, d) is complete.

(2) f and g are weakly increasing mappings with respect to \preccurlyeq .

(3) f is continuous.

(4) f is an almost generalized (ψ, ϕ) -contractive mapping with respect to g , that is,

$$\psi(d(fx, fy)) \leq \psi(M(x, y)) - \phi(M(x, y)) + L\psi(N(x, y))$$

for $x, y \in X$ with $x \preccurlyeq y$ and $L \geq 0$, where

$$M(x, y) = \max \left\{ d(x, y), d(x, fx), d(y, gy), \frac{d(x, gy) + d(y, fx)}{2} \right\}$$

and

$$N(x, y) = \min\{d(x, fx), d(y, fy), d(x, gy)\}.$$

Proof. The proof of (1) is clear. To prove (2), let $x \in X$. If $x \in \{0, 1, 2\}$, then $fgx = 0 \leq gx = 0$ and $gfy = 0 \leq fy$. So $gx \preccurlyeq fgx$ and $fy \preccurlyeq gfy$. While, if $x \geq 3$, then $fgx = x - 3 \leq x - 2 = gx$ and $gfy = x - 3 \leq x - 1 = fy$. So $gx \preccurlyeq fgx$ and $fy \preccurlyeq gfy$. Hence f and g are weakly increasing mappings with respect to \preccurlyeq . To prove that f is continuous, let (x_n) be a sequence in X such that $x_n \rightarrow x \in X$. By the definition of the metric d , there exists $k \in \mathbb{N}$ such that $x_n = x$ for all $n \geq k$. So $fx_n = fx$ for all $n \geq k$. Hence $fx_n \rightarrow fx$ that is, f is continuous. To prove (4), given $x, y \in X$ with $x \preccurlyeq y$. So $y \leq x$. Thus, we have the following cases:

Case 1. $x \in \{0, 1\}$ and $y \in \{0, 1\}$. Then $d(fx, fy) = 0$ and hence

$$\psi(d(fx, fy)) \leq \psi(M(x, y)) - \phi(M(x, y)) + L\psi(N(x, y))$$

for each $L \geq 0$.

Case 2. $x, y \geq 2$ and $x = y$. Then $d(fx, fy) = 0$ and hence

$$\psi(d(fx, fy)) \leq \psi(M(x, y)) - \phi(M(x, y)) + L\psi(N(x, y))$$

for each $L \geq 0$.

Case 3. $x > y \geq 2$.

If $x = y + 1$, then

$$d(fx, fy) = d(fx, f(x-1)) = d(x-1, x-2) = 2x-3,$$

and

$$\begin{aligned} M(x, y) &= M(x, x-1) \\ &= \max \left\{ d(x, x-1), d(x, fx), d(x-1, g(x-1)), \frac{d(x, g(x-1)) + d(x-1, fx)}{2} \right\} \\ &= \max \left\{ 2x-1, 2x-4, \frac{2x-3}{2} \right\} = 2x-1. \end{aligned}$$

Since

$$(2x-3)^2 \leq (2x-1)^2 - (2x-1),$$

we have

$$\psi(d(fx, fy)) \leq \psi(M(x, y)) - \phi(M(x, y)) + L\psi(N(x, y))$$

for each $L \geq 0$.

If $x > y + 1$, then

$$d(fx, fy) = d(x-1, y-1) = x+y-2$$

and

$$\begin{aligned} M(x, y) &= \max \left\{ d(x, y), d(x, fx), d(y, gy), \frac{d(x, gy) + d(y, fx)}{2} \right\} \\ &= \max \left\{ x+y, 2x-1, 2y-2, \frac{2x+2y-3}{2} \right\} = 2x-1. \end{aligned}$$

As

$$(x+y-2)^2 \leq (2x-3)^2 \leq (2x-1)^2 - (2x-1),$$

we have

$$\psi(d(fx, fy)) \leq \psi(M(x, y)) - \phi(M(x, y)) + L\psi(N(x, y))$$

for each $L \geq 0$. By combining all cases together, we conclude that f is an almost generalized (ψ, ϕ) -contractive mapping with respect to g . Thus f, g, ψ and ϕ satisfy all the hypotheses of Theorem 2.3 and hence f and g have a common fixed point. Indeed, 0 is the unique common fixed point of f and g . \square

3 Applications

Let Φ denote the set of functions $\phi : [0, +\infty) \rightarrow [0, +\infty)$ satisfying the following hypotheses:

- (1) Every $\phi \in \Phi$ is a Lebesgue integrable function on each compact subset of $[0, +\infty)$,
- (2) For any $\phi \in \Phi$ and $\varepsilon > 0$, $\int_0^\varepsilon \phi(s)ds > 0$.

It is an easy matter, to check that the mapping $\psi : [0, +\infty) \rightarrow [0, +\infty)$ defined by

$$\psi(t) = \int_0^t \phi(s) ds$$

is an altering distance function. Therefore, we have the following results.

Corollary 3.1. *Let (X, \preceq) be a partially ordered set having a metric d , such that the metric space (X, d) is complete. Let $f : X \rightarrow X$ be a non-decreasing continuous mapping with respect to \preceq . Suppose that there exist $k \in [0, 1)$ and $L \geq 0$ such that*

$$\int_0^{d(fx, fy)} \phi(s) ds \leq k \int_0^{\max\{d(x, y), d(x, fx), d(y, fy), \frac{1}{2}(d(x, fy) + d(y, fx))\}} \phi(s) ds + L \int_0^{\min\{d(x, fx), d(y, fy)\}} \phi(s) ds$$

for all comparable $x, y \in X$. If there exists $x_0 \in X$ such that $x_0 \preceq fx_0$, then f has a fixed point.

Proof. Follows from Corollary 2.1 by taking $\psi(t) = \int_0^t \phi(s) ds$. \square

Corollary 3.2. *Let (X, \preceq) be a partially ordered set having a metric d , such that the metric space (X, d) is complete. Let $f, g : X \rightarrow X$ be two weakly increasing mappings with respect to \preceq . Suppose that there exist $k \in [0, 1)$ and $L \geq 0$ such that*

$$\int_0^{d(fx, gy)} \phi(s) ds \leq k \int_0^{\max\{d(x, y), d(x, fx), d(y, gy), \frac{1}{2}(d(x, gy) + d(y, fx))\}} \phi(s) ds + L \int_0^{\min\{d(x, fx), d(y, gy)\}} \phi(s) ds$$

for all comparable $x, y \in X$. If there exists $x_0 \in X$ such that $x_0 \preceq fx_0$, then f has a fixed point.

Proof. Follows from Corollary 2.3 by taking $\psi(t) = \int_0^t \phi(s) ds$. \square

Finally, let us finish this article by noticing the following remarks:

Remark 1. Theorem 2.1 of [26] is a special case of Corollary 2.1

Remark 2. Theorem 2.2 of [26] is a special case of Corollary 2.2

Remark 3. Theorem 2.3, Corollaries 2.4 and 2.6 of [26] are special cases of Corollary 2.3

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All the authors contributed equally. All authors read and approved the final manuscript.

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